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# Study the Characteristics of Robust PID Controller for Time Delay System

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**Abstract**— In practical field like engineering, biological and other control systems, which deals with a robust PID controller, time delay is always kept in consideration. While controlling a process various uncertainties may arise, which can be resolved by using a feedback system to obtain a desired result. Due to application of several Sensors & Transducers, time delay may occur which reduces the performance of the system. The time delay may be also generated due to communication delay of various parts of system. System oscillation or instability may arise in the applied system due to time delays generated in the controllers, which may cause uncertainties in the system.

That's why a lot of literatures presented on time delayed processes for system stabilizing, analysis of stability and controlling time delay system are available. In the first order controller, controlling method is very simple and it can easily be implemented practically as these lower order controllers can reduce the controller complexities, so we can stabilize a higher order process or complex plant by using various lower order controllers. While designing a controller, system stabilization is the initial requirement which stabilizes all the PID parameters like unit step response, peak overshoot, rise time, settling time etc. Some alterations are made in the process plant where we can apply our proposed method to any first order controller to obtain a stabilized output. As an outcome, the result of this research work can also be applied for any experimental or industrial applications.

**Index Terms**— PID controller, Linear time invariant (LTI) system, Delay Time, Stability Region, Robustness, SCADA system.

## 1 Introduction

In control theory, a controller is a device, used in electrical or mechanical field in association with hydraulic, pneumatic or electronic techniques in combination, but more recently, in the form of a microprocessor or computer, which monitors and physically alters the operating conditions of a given dynamical system.

A PID (Proportional Integral Derivative) controller is the most commonly used instrument in industrial control applications, which can be used for regulation of speed, temperature, flow, pressure and other process variables. Field mounted PID controllers can be placed close to the sensors or the regulating devices can be monitored centrally using a SCADA system, e.g. a temperature Controller using a Digital PID controller.

PID controller shows robust stability in the real industrial process. Classic PID controller is better suited to precise mathematical models having poor self-adaptive nature and it is hard to obtain robust stability region which is caused by the external interruption and uncertainty.

As we know that almost all practical working methods include uncertainties so, designing robust modified PID controller with uncertainties is more important. Hence, in this paper we have explained the characteristics of Robust PID controller for a Time Delay System.

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## 2 THEORETICAL BACKGROUND

A PID controller (sometimes called a three term controller) reads the sensor signal, normally from a thermocouple or RTD, and converts the measurement to engineering units e.g. Degrees C. It then subtracts the measurement from a desired set-point to determine an error. The error is acted upon by the three (P, I & D) terms simultaneously:

**Proportional (Gain)**

The error is multiplied by a negative (for reverse action) proportional constant P, and added to the current output. P represents the band cross over such that a controller's output is proportional to the error of the system e.g. for a heater, a controller with a proportional band of 10ºC and a set-point of 100ºC would have an output of 100% up to 90ºC, 50% at 95ºC and 10% at 99ºC. If the temperature overshoots the set-point value, the heating power would be cut back further. Proportional only control can provide a stable process temperature but there will always be an error between the required set-point and the actual process temperature.

**Integral (Reset)**

The error is integrated (averaged) over a period of time, and then multiplied by a constant I, and added to the current control output. Here I represents, the steady state error of the system and will remove set-point / measured value errors. For many applications Proportional + Integral control will be satisfactory with good stability and at the desired set-point.

**Derivative (Rate)**

The rate of change of the error is calculated with respect to time, multiplied by another constant D, and added to the output. The derivative term is used to determine a controller's response to a change or disturbance of the process

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temperature (e.g. opening an oven door). The larger the derivative term, the more rapidly the controller will respond to changes in the process value.

**Tuning of PID Controller**

The parameters P, I and D need to be "tuned" to suit the dynamics of the process being controlled. Any of the terms described above can cause the process to be unstable, or very slow to control, if not correctly set. These days temperature control using digital PID controllers have automatic auto-tuned functions. During the auto-tuning period, a PID controller controls the power to the process and measures the rate of change, overshoot and response time of the plant. This is often based on the **Zeigler-Nichols method** of calculating controller term values. Once the auto-tune period is completed the P, I &D values are stored and used as the PID controller.

## 3 REVIEWS

There is a growing demand of PID in diverse application areas, such as Plant Control, Flow Control, Speed Control, Navigation etc.

N. Hohenbichler et al. Presented a “Synthesis of Robust PID Controllers For Time Delay System”. This research uses the “Parameter Space Approach”. International Research Journal of Engineering and Technology (IRJET).

Yifei YANG et al. Presented a Robust Stability Regions of PID Parameters for Uncertainty Systems with Time Delay Using D-partition Technique.

Takaaki Hagiwara et al. In his paper he provided a method for uncertain time delayed plants that stabilize the modified PID controller at a great extent. It creates the time delay plant stable and find reliable values of P, I and D parameters that do not depends each other. It also gives heat flow experimental result for illustrating this method.

P. V. Gopi Krishna Rao et al. Here the tuning method given is Model based. For robust operation of controller a method named IMC-PID (Internal Model Control tuning method) is used. First Order plus Delay Time (FOPDT) model can easily characterize the process dynamics for implementing the IMC in large industrial applications.

Karim Saadaoui et al. Another class of stabilized time delay system is presented in this paper. Many physical applications can use this method that may be locating any ship or other under water vehicle. Linear time invariant delay free systems can be modelled with this proposed method.

## 4 METHODOLOGY

Block diagram below represents a control system with a closed loop consisting of a controller and a process or plant in the absence of any uncertainty and time delay, or we can say in ideal condition. Here Gp(s) shows the nominal plant and K(s) is the PID controller. r(s) and y(s) are input signal and output signal respectively. The input to the system is the “set- point” i.e. the desired output. The input given to the controller is the error value that we earlier calculated.

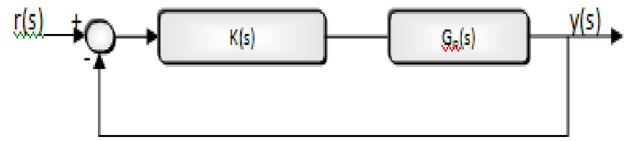

Fig 1: Block diagram of system without delay and uncertainly.

A PID controller as name suggests consists three elements namely a proportional element, an integral element and a derivative element, all three connected in parallel. All of them take error as input Kp, Ki and Kd are the gains of P, I, D elements respectively.

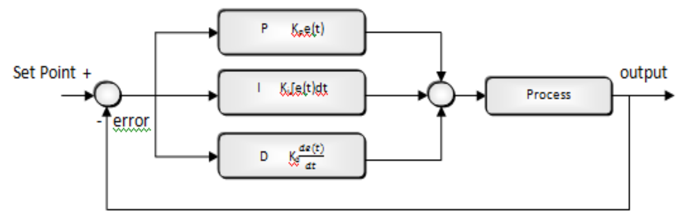

Fig 2: Block diagram of PID Controller

The synthesis step is extended to time delay systems, but important results for the practical application are still missing. Also, the analysis step is not developed in the literature and results have not been compared with existing tuning methods.

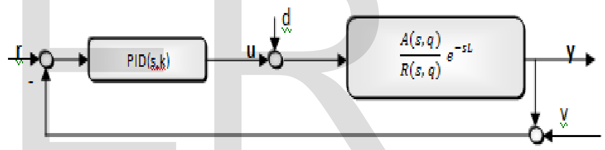

Fig 3: Single Loop with PID controller and time delay system.

Now, we will discuss the mathematical formulations that are most vital in order to obtain the set of PID controller gains that will enable us to obtain the nominal stability boundary and robust stability region for an arbitrary order perturbed plant with additive uncertainty, while ensuring closed loop stability.

$$K(s) = K_p + \frac{Ki}{s} + K_d s$$

A SISO and linear time invariant (LTI) system with additive uncertainty is shown. Here Gp(s) is the nominal plant, K(s) is the PID controller, and WA(s) is the additive weight. The input signal and the weighted output signal are r(s) and y(s) respectively.

In Figure, GΔ(s) represents the perturbed plant which includes ΔA(s), which is any stable transfer function such that $|\Delta_a(j\omega)| \leq 1, \forall\omega|$

$$G_p(j\omega) = Re(\omega) + jIm(\omega)$$

$$K(j\omega) = K_p + \frac{Ki}{j\omega} + K_d j\omega$$

$$W_A(j\omega) = A_A(\omega) + jB_A(\omega)$$

In the frequency domain we can represent these transfer functions as,

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In order to achieve robust stability for the perturbed system, we want to find all PID controller gains that stabilize the closed loop system for the entire range of uncertainties. This goal can be achieved if the nominal system is stable and the robust stability constraint,

$$\|W_s(j\omega)K(j\omega)S(j\omega)\|_\infty \leq \gamma$$

is satisfied, where S(jω) is the sensitivity function and $\gamma = 1$.
Here shows the region of convergence or boundary condition for controller beyond that the controller cannot operate correctly or controller fails.

## PROBLEM FORMULATION

Consider a single loop PID controller and a linear time delay system as shown in figure, given by the transfer functions:

$$PID(s, k) = \frac{K_I + K_P s + K_D s^2}{s(1+T_R s)}, \qquad (1)$$

$$G(s, L, q) = \frac{A(s,q)}{R(s,q)} e^{-sL}, \qquad (2)$$

In which, k = ( $K_I, K_P, K_D$)T are the controller parameters. (TR confirms feasibility of the controller and filters noise v; it is assumed to be fixed prior to the controller design, e.g. by adding a non-dominant pole to R(s, q)). The unknown but constant plant parameters are the dead time L > 0 and the parameters in the vector q. They lie in an operation domain:

$$Q = \{ (L, q)^T \mid L \in [L^-; L^+], q_i \in [q_i^-; q_i^+] \}, \quad (3)$$

Where $q_i^-$, $q_i^+$ are specified as the lower and upper limits of parameter $q_i$ in **q** (analog $L^-$ and $L^+$).

The problem may arise in designing a robust PID controller is to find a set of controller parameters k = k*, that meets the specification for all values of (L, q)T ∈ Q. Specifications can be assumed in the form of Hurwitz stability (all the roots of the characteristic function are situated in the open left half plane (LHP)) and σ-stability (all roots have a real part smaller then a real number $\sigma_0$). The characteristic function of the loop in Fig.

$$P(s, k, L, q) = (K_I + K_P s + K_D s^2)\, A(s, q) + s(1 + T_R s)\, R(s, q)\, e^{sL} \quad (4)$$

Here, $s(1 + T_R s) R(s, q) = B(s,q)$

Including polynomials are,

$$A(s, q) = a_0(q) + a_1(q)\, s + \ldots + a_m(q)\, s^m, \qquad (5)$$
$$B(s, q) = b_0(q) + b_1(q)\, s + \ldots + b_n(q)\, s^n, \qquad (6)$$

With $a_m(q) \neq 0$, $b_n(q) \neq 0$, belongs to the class of quasi polynomials due to the dead time. (Note that $b_0$(q) = 0 for basic case of a PID controller. However, later a $b_0(q) \neq 0$ may appear through transformations.) The principal term condition requires for Hurwitz stability that in the case of PID control $K_D \neq 0$ the degrees fulfil n ≥ m+2. In the sequel we treat only this case (i.e. we assume a proper A(s, q) / R(s, q) for $T_R \neq 0$.

## PARAMETER SPACE APPROACH

The parameter space approach can be used to solve the problem in two main steps.

The controller synthesis step: We compute the stable (either Hurwitz or σ-stable) region in the space of controller parameters k for several representatives (L*, q*)T out of Q (usually the vertices). A candidate for a robust controller $K^*$ is chosen from the intersection of stable regions. This controller satisfies the specification for the representatives.

The second step,
The control loop analysis, is applied to test the robust stability for the continuum of all values in Q. Now we compute the stable region in the space of plant parameters (L, q)T with fixed controller k*. If Q lies entirely in the stable region, then a solution of the problem is found. The calculation of a Hurwitz stable region in a parameter space (either k or (L, q)T ) is based on the fact that the roots of the quasi polynomial with continuous coefficient functions ai (q), bi (q) do not jump when the parameters are changed continuously. Thus, a stable quasi polynomial, whose roots all lie in the LHP, becomes unstable if and only if at least one root crosses the imaginary axis. The parameter values of the root crossings form the stability boundaries in the parameter space, which can be classified into three cases: the real root boundary (RRB), where a root crosses the imaginary axes at the origin (substitute s = 0 in the quasi polynomial), the infinite root boundary (IRB), where a root leaves the LHP at infinity (set |s|→ ∞) and the complex root boundary (CRB), where a pair of conjugate complex roots crosses the imaginary axes (substitute |s| = j and sweep over all real > 0). These stability bo undaries separate different regions in the parameter space. To classify a region as Hurwitz stable it suffices to prove stability for one inner test point (e.g. by the Nyquist criterion).

## CONTROLLER DESIGN ALGORITHM

The proposed controller design procedure is summarized in the following steps:
1. Specify the maximum real part σ from closed-loop settling time requirements.
2. Compute the σ-stable regions in controller parameter space for representatives (usually the vertices) of the Q-domain.
3. Determine the intersection of the σ-stable regions.
4. Choose a candidate controller out of the intersection.
5. Compute the σ-stable region in plant parameter space for the candidate controller.
6. If the Q-domain lies entirely in the σ-stable region, then the problem is solved. σ-stability can be reduced to the Hurwitz case by the substitution s = v + $\sigma_0$which leads to a transformation in parameters and polynomials. So Hurwitz stability is considered first in the next paragraphs.

## Controller Synthesis

For each fixed representative (L*, q*)T the Hurwitz stability boundaries of in the k-space are determined. The RRB turns out to be simply a straight line given by the equation:

$$P(0, k) = K_I A_0 + B_0 = 0 \quad \leftrightarrow K_I = -b_0 a_0$$

(In the basic case we have $b_0 = 0$ and the RRB is $K_I = 0$.)

More theoretical difficulties arise when calculating the IRB. Quasi polynomials possess an infinite number of roots, which can not be calculated analytically in the general case. However, the asymptotic location of roots far from the origin. It turns out that infinite root boundaries only exist, if the degree equation n = m + 2 is fulfilled (in case of $K_D \neq 0$.). These are two straight lines

$$K_D = \pm (b_n / a_n)$$

The calculation of the CRB starts analog to the delay free case of polynomials. The root condition P(j, k) = 0 can be separated into a system of two equations for real and imaginary part.

$$\begin{pmatrix} R_P(\omega,k) \\ I_P(\omega,k) \end{pmatrix} = \begin{pmatrix} R_A & -\omega^2 R_A \\ I_A & -\omega^2 I_A \end{pmatrix} \begin{pmatrix} K_I \\ K_D \end{pmatrix} + \begin{pmatrix} R_B - K_p \omega I_A \\ I_B + K_p \omega R_A \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

where Ñ(j$\omega$) = B(j$\omega$) $e^{j\omega L^*}$ and R, I denote the real and imaginary parts of A, Ñ and P at (j$\omega$). Clearly, the matrix multiplying $(K_I, K_D)$T is singular. Thus, the key idea is to fix $K_P = K_P$* and to evaluate the CRB in the ($K_D$, $K_I$) –plane. A solution exists and only exists for the real zeros $\omega_{gi}$ of

$$g(\omega) = \det \begin{pmatrix} R_A & R_{\tilde{N}} - K_P^* \omega I_A \\ I_A & I_{\tilde{N}} + K_P^* \omega R_A \end{pmatrix}$$

$$= \omega K_P^* \left( R_A^2 + I_A^2 \right) + R_A I_{\tilde{N}} - I_A R_{\tilde{N}}$$

The zeros of g($\omega$) are called singular frequencies. For each i appears a straight line as CRB in the ($K_D$,$K_I$ )-plane, ruled by the equation.

$$K_I = \omega_{gi}^2 K_D + K_I^0(\omega_g),$$

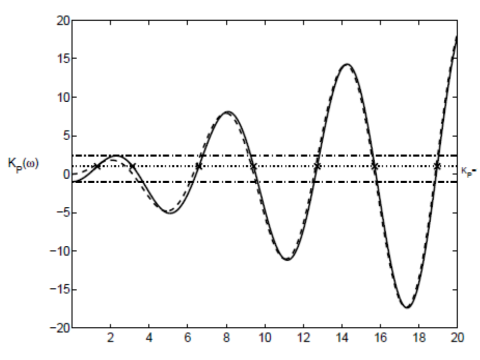


Figure: Function $K_P$ ($\omega$) (solid), its limit function (dashed), singular frequencies for $K_P^*$ = 1 (x-marks) and stabilizing $K_P$ -interval (dash-dotted) of $G_1$(s).

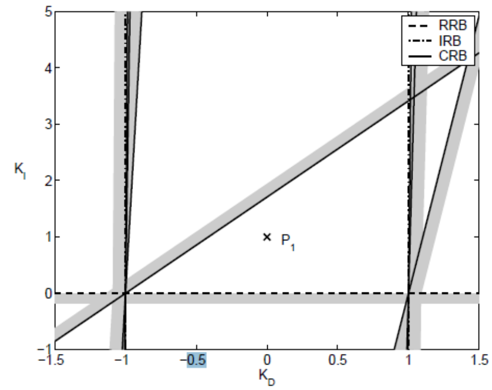


Figure: Stability boundaries in ($K_D$, $K_D$)-p lane for $K_P^*$ = 1 of $G_1$(s). The side of the lines with more unstable poles is shaded.

Where $K_I^0(\omega_g)$ can be easily determined. Thus, the stability boundaries RRB, IRB and CRB are straight lines in the ($K_D$, $K_D$)-plane and partition the plane into convex polygons. (Additionally, for each boundary line the side can be determined which possesses the lower number of stable poles.) The singular frequencies may be determined by a graph of

$$K_P(\omega) = \frac{-R_A I_{\tilde{N}} + I_A R_{\tilde{N}}}{\omega(R_A^2 + I_A^2)}$$

Graphically, the singular frequencies for a fixed $K_D$are the abscissa values of the intersections between the $K_P(\omega)$-plot and the ($K_P$=$K_D$)-line. Due to the dead time the number of singular frequencies is infinite. Algorithms for the automatic calculation of the singular frequencies can be found. The function $K_P(\omega)$ and the resulting boundaries in the ($K_D$, $K_P$)-plane for a fixed $K_P^*$= 1 are demonstrated for the example system (with ideal PID controller $T_R$ = 0)

$$G_1(s) = \frac{1}{s+1} e^{-s}$$

## PID design using σ-stability

The concept of σ -stability can be used to speed up the transient responses robustly. Concerning the synthesis step, this case can be reduced to the Hurwitz case by substituting s = v +$\sigma_0$. Following transformations result

$$K_0' = K_I + K_P \sigma_0 + K_D \sigma_0^2$$
$$K_P' = K_P + 2 K_D \sigma_0$$
$$K_D' = K_D$$
$$A'(v) = A(v + \sigma_0)$$

Following the parameter space approach presented in section, the smallest possible value for $\sigma_0$ is determined for a given system and an operation domain, s. t. the intersection of the stable regions belonging to the vertices of Q is not empty. This $\sigma_0$ is approximated by an iterative approach: Beginning with zero $\sigma_0$ is stepwise reduced and the function $K_P(\omega)$is plotted

for each vertex, until the work hypothesis reveals that there is no interval of $K_P$ that stabilizes simultaneously all vertices. With the last $\sigma_0$ having such a interval, the stable k-regions are computed for all vertices, and a controller $K^*$ is taken out of the intersecting region. The analysis step can be reduced to the Hurwitz case, if the second parameter q enters in form of a dc-gain into the quasi polynomial.
In that case, the transformations are

$$q'' = q e^{\sigma_0 L} \qquad D''(v) = D(v+\sigma_0)$$

$$L'' = L \qquad B''(v) = B(v+\sigma_0)$$

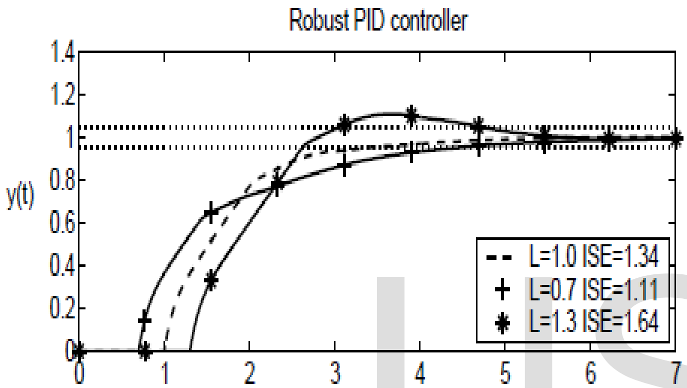


Figure: Step responses of PID and IMC controlled loop of $G_2$(s, L) for nominal, maximal and minimal L.

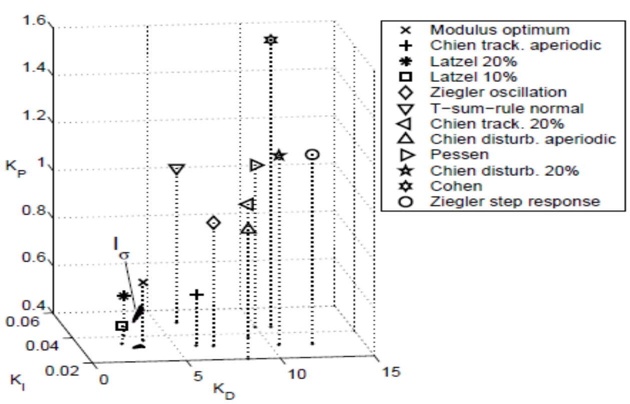


Figure: Intersection ($I_\sigma$) of the -stable regions ($\sigma_0$=−0.05) of the vertices of $Q_4$ for $G_4$(s, L, K) and tuning rule controllers. The tuning rules are sorted by the settling time after a reference step for the nominal plant.

## Conclusion

The parameter space approach offers convincing results in the synthesis of robust PID controllers for time delay systems. The modified tuning method is systematic, universal and transparent and leads to superior or similar results than literature examples.

Exact stability (Hurwitz or σ-stability) regions can be determined in the space of controller and plant parameters while treating the dead time without approximation.
The development of an interactive graphical software package based on the stated algorithm seems very promising to be a helpful tool in daily engineer's work. So an engineer would be able to re-tune the huge amount of existing PID loops at low cost in industry. This PID controller method can improve the relative stability and improves the steady state for a system . The simulation result can be observed as graphically as well as the related values. By applying some variation in algorithm the robust PID controller has made the system stabilized through various types of delay present in the system. The robustness of the controller is the major advantage and it guarantees the robustness of system with respect to plant communication variation and disturbance caused by the external factors.
The simulation result shows the controller gives good time response as well as reduces the delay time. PID controller receives the sensor information or transmits its output through the communication network.
This PID controller method can improve the relative stability and improves the steady state. The simulation result can be viewed as graphically as well as the related values.

## Future Scope

Second-order integrating processes with time delay processes can be extend by using the analytical design method of PID controller.
The other methods can be developed for PID controllers by using H- infinity optimal criterion that can be applicable in chemical or industrial second-order plants that may or may not have time delays.

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